

Checkers with Runway 13

Aryan Ayyar

Kai Tak's Runway 13 presented an unusually visible engineering problem: its instrument localiser and runway differed by 47 degrees, leaving a large aircraft to turn over Kowloon, descend, align and stop. This investigation follows that problem from the lift vector to the end of the runway, combining dated survey maps, aerial photogrammetry and the official CAL605 accident report with an explicit mathematical landing model. A worked turn shows how 15 degrees of bank at 78 m/s can accumulate the direction change in about 24.35 seconds. The complete calculation reveals a second constraint: touchdown commits the aircraft to delay and braking distances that depend strongly on ground speed. Constructed calm-air and steady-wind landings leave 254.68 m and 552.77 m respectively under the same assumed braking law; separate interval-arithmetic certificates verify their constraints continuously in time. Historical stereo images recover three-dimensional surface structure while exposing the limits of local control and incomplete coverage. CAL605's rollout record shows why reaching the runway and completing the landing are different engineering tasks. The trajectories use a reduced, uncalibrated aircraft model and synthetic approach geometry; the two optimized scenarios are distinct constructions, not a causal wind experiment or a reconstruction of the historical flight.

Version 5 · 13 October 2026

Independent research white paper. Not peer reviewed or operational flight guidance.

Author's foreword

FIRST-PERSON DRAFT FOR ARYAN AYYAR'S REVIEW

My interest in Kai Tak began with a childhood fascination with aircraft. A large passenger aeroplane turning above Hong Kong seemed to bring two very different scales into the same scene: the machine in flight and the streets below it. I wanted to understand how they fitted together.

Richard Hammond's *Engineering Connections* encouraged that curiosity. Understanding a machine's workings made it more absorbing to watch. The imagery and music of *Enter the Dragon* were another part of my early interest in Hong Kong. Later, studying at Hong Kong Polytechnic University gave the city a place in my own life. My time there was after Kai Tak had closed; the airport I arrived at was Chek Lap Kok.

Hong Kong matters to me personally, beyond this research question. The question itself is strong enough: how could an aircraft of this size turn towards that runway and still complete its landing?

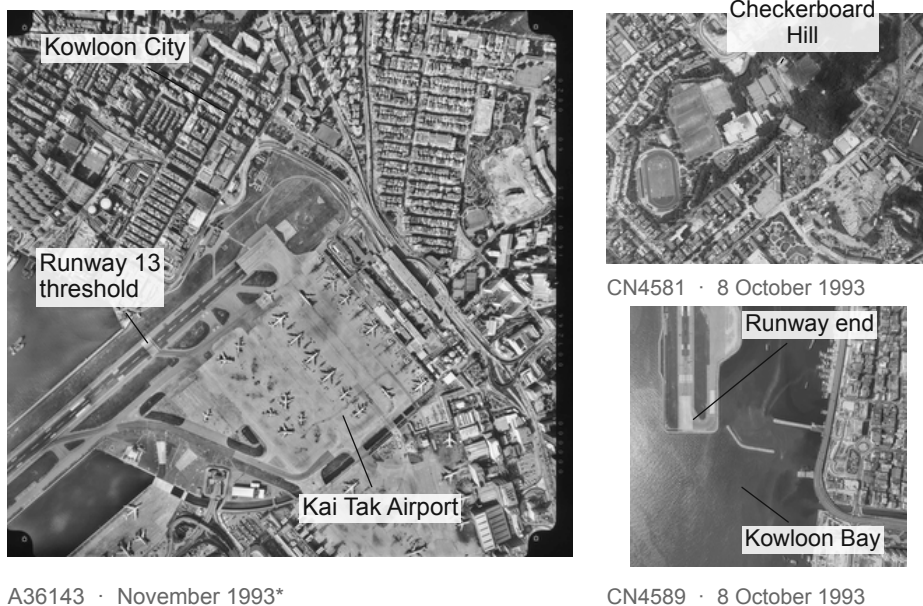
I approach it as a financial economist and mathematician. The tools are familiar; the engineering invites careful learning. Following the turn leads to forces, speed and descent. Following the aircraft to a full stop brings the runway into the calculation. Returning to old photographs brings the city back into view. That movement between a place, a physical question and a mathematical explanation is what drew me to the project, and what I hope will draw a reader through it.

1. A runway in a different direction

At Kai Tak, the radio beam that supplied lateral approach guidance did not point down the runway. This localiser pointed along 088° magnetic. Runway 13 lay along 135° . Between the two directions stood the problem that made the airport famous: a large passenger aircraft had to make a substantial visual turn, descend, align over the pavement and arrive with enough room to stop (Hong Kong Civil Aviation Department 1995, §§1.8.1, 1.10).

The setting made the geometry hard to ignore. Dense neighbourhoods crowded the landward side of the airport; hills rose beyond them; the runway extended into Kowloon Bay. From the city, the final turn looked improbably close. On a plan, it becomes a question about space and time. How much bank is needed to accumulate the change of direction? What height and speed can be carried through it? And, after the aircraft reaches the runway, how much of the landing remains?

THE CITY AT THE THRESHOLD



Separate source views; original scan perspectives.

*Catalogue: 2 November. Handwritten frame label: apparently 3 November.

Figure 1

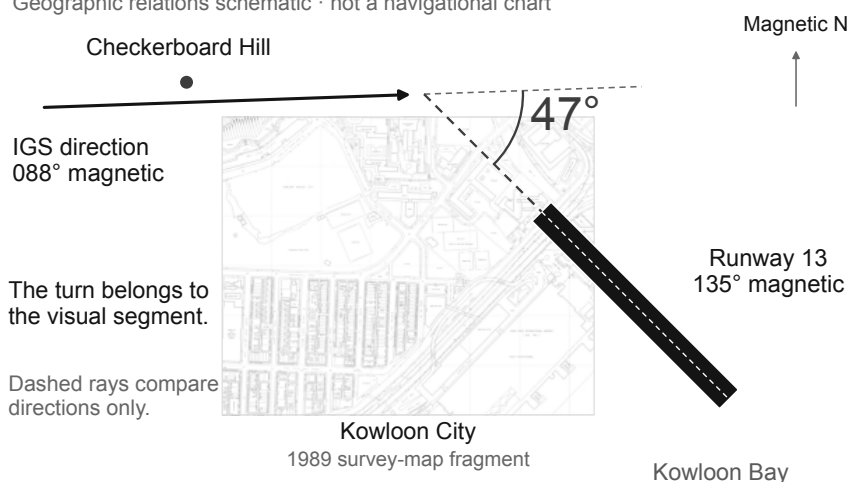
Kai Tak and the city beside it in three separate Lands Department aerial frames, not a stitched panorama. CN4581 and CN4589 are dated 8 October 1993. A36143 is catalogued as 2 November 1993; its handwritten label appears to read 3 November, an unresolved discrepancy. Fine leaders locate visible features without inserting a flight path. The photographs retain their scan perspectives; no metric scale is inferred. Source: Lands Department, HKSAR Government; CSDI Portal and DATA.GOV.HK, DAP-L0. Government intellectual-property ownership retained; supplied as-is, without endorsement.

The two published bearings differ by 47° . They share a magnetic reference, so subtraction is legitimate. But the result describes directions in a horizontal plan. It does not say that the aircraft's wings must bank by 47 degrees. Bank is the inclination of the wings; a change of direction is the accumulated result of turning over time. Confusing the two makes the approach look like a much more violent manoeuvre than the number actually specifies.

The published geometry also separates two kinds of guidance. Kai Tak's Instrument Guidance System (IGS), introduced in 1974, supplied a localiser and a 3.1° glidepath (Hong Kong Civil Aviation Department n.d., 1995). It brought an aircraft towards the visual segment. The pilot then had to make the right turn towards Runway 13, using the visible setting and the procedure. The instrument system had not bent its beam around the city.

TWO DIRECTIONS IN ONE LANDSCAPE

Geographic relations schematic · not a navigational chart



47° is a change of direction, not a wing-bank angle.

Figure 2

The offset that defines the approach: 088° magnetic for the IGS localiser and 135° magnetic for Runway 13, a difference of 47°. Geographic context locates the problem; the bearing construction compares published directions, not a surveyed turn point. Magnetic bearing, ground track and air heading are distinct; the latter two separate in wind. Sources: CAD Report 1/95, §§1.8.1 and 1.10, Appendix 1; historical geographic sources identified in the figure provenance. Map fragment, landmark and runway symbol are schematic, not spatially registered.

The middle marker lay a reported 1.7 nautical miles from touchdown. If visual flight had not been achieved there, the procedure required a missed approach (Hong Kong Civil Aviation Department 1995, §1.8.1). Visibility therefore belonged to the geometry of the task: the celebrated final turn was a visual manoeuvre, not an instrument-guided turn to be completed without sight of the landing environment. Checkerboard Hill supplied a visual reference in Kowloon, and the CAD's retrospective account places the right turn over Kowloon City (Hong Kong Civil Aviation Department 2025).

A plan view can establish the directional mismatch. It cannot yet say whether the aeroplane has enough room. That depends on how quickly its velocity can turn and on where the turn begins and ends. Nor does alignment alone finish the approach. At the runway the aircraft must have a suitable height, descent rate and speed, all at the same time.

The final distance has a specific historical definition. CAD Report 1/95 gives a runway length of 3,331.5 m, a width of 61 m and a landing distance available of 2,786 m (Hong Kong Civil Aviation Department 1995, §1.10). The last quantity is the one used in the threshold-relative stopping calculations below. The longer lengths reported in general airport histories describe a different

quantity or historical context; Appendix G retains those distinctions. A landing cannot spend pavement that its distance convention has already excluded.

By the time Kai Tak closed in July 1998 and Hong Kong's airport moved to Chek Lap Kok, this arrangement had become one of aviation's most recognizable scenes (Hong Kong Civil Aviation Department 2025). Its mechanics begin with a less dramatic image: the lift vector of a steadily banked aircraft.

2. How a modest bank accumulates a turn

In straight, level flight, lift supports the aircraft's weight. Tilt that lift vector by banking the wings and it acquires a horizontal component. The horizontal component bends the velocity; the vertical component must continue to support the aircraft. A turn is the simultaneous performance of those two jobs.

For a steady, coordinated, level turn in still air, write lift as $\mathcal{L}$, mass as m , airspeed as V , bank angle as ϕ and turn radius as R . Resolving forces vertically and towards the centre of the turn gives

$$\mathcal{L} \cos \phi = mg, \quad \mathcal{L} \sin \phi = \frac{mV^2}{R} \implies R = \frac{V^2}{g \tan \phi}. \quad (1)$$

Dividing the two force equations removes both mass and lift. The resulting relation describes the geometry required by speed and bank. An actual aircraft still has to generate the lift: mass continues to matter for lift capacity, stall and structural demands.

Now put 78 m/s and 15° into the equation. The radius is approximately 2.32 km. That may initially sound too large for a manoeuvre so often described through images of nearby buildings. But a radius is not the length of the arc that must be flown, and the aircraft is not completing a circle. For a direction change $\Delta\psi = 47\pi/180$ radians, the arc length is $R\Delta\psi$, approximately 1,899 m. At the stipulated speed it takes 24.35 seconds.

Fifteen degrees of bank does not produce fifteen degrees of total turn. It produces a turning rate. Here the rate persists long enough to accumulate the much larger change in direction. In the ideal model, $\dot{\psi} = g \tan \phi / V$; the final angle is its integral over time. The bank angle and the angle turned live in different parts of that calculation.

The load factor offers a second check on intuition. Vertical force balance requires $\mathcal{L}/(mg) = 1/\cos \phi$. At 15° it is 1.0353. In this benchmark, the lift magnitude is only about 3.5% greater than weight. The considerable change in direction comes from sustaining a modest inward acceleration over time, rather than from an instant of extreme force.

Speed alters the bargain. At fixed bank, radius grows as V^2 : moving faster requires a wider circle. Yet the time for a fixed direction change grows only linearly with V , because the greater arc length is traversed at the greater speed. These are two views of the same motion, and both matter to a runway approach.

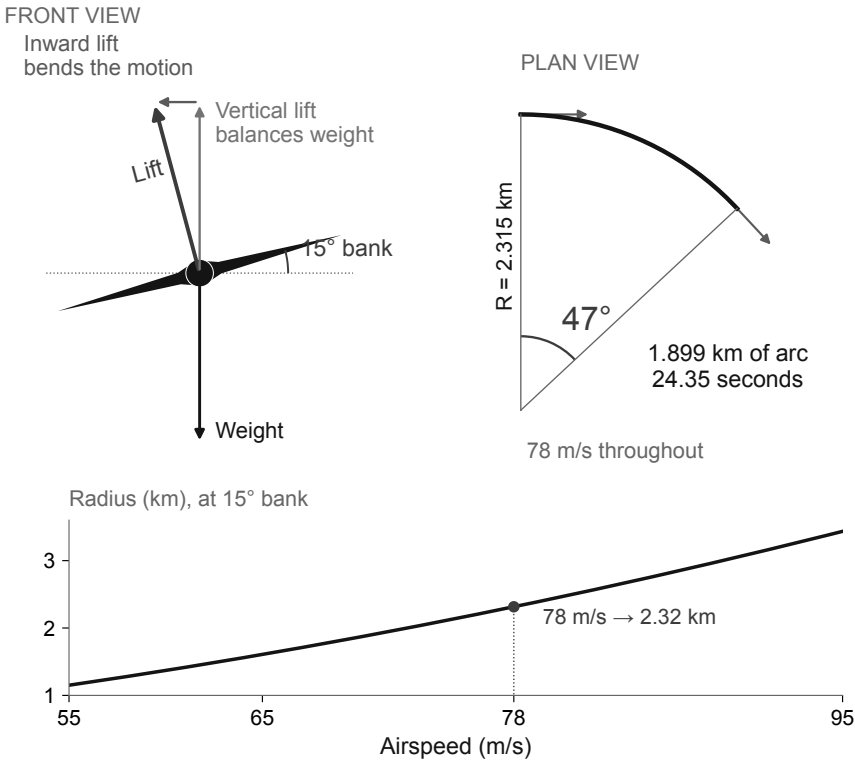


Figure 3
A banked lift vector supports weight vertically and supplies the inward acceleration horizontally. The plan-view arc accumulates 47° of direction change at a constant 15° bank. At 78 m/s, equation (1) gives a 2.315 km radius, 1.899 km of arc and 24.35 s of turning time, using $g = 9.80665 \text{ m/s}^2$. This ideal coordinated, level, still-air benchmark excludes roll-in and roll-out; it is not spatially fitted to the historical approach.

A turn that can be completed quickly may still occupy too much space; a narrow turn may demand more bank than the permitted envelope allows.

An aeroplane also needs time to roll into the bank and out again. During those transitions the turning rate changes. For the complete landing, speed, bank and descent change together, so the circular arc becomes an explanatory benchmark rather than a prescribed route.

One requirement is still absent from the diagram. The aircraft has to lose height. It must reduce its downward motion before contact and bring an acceptable forward speed onto the runway. The turn has used almost two kilometres of motion in our example; the next calculation asks what the rest of that motion must accomplish.

3. The landing continues beyond touchdown

At 78 m/s, a straight still-air descent along a 3.1° slope has a vertical velocity of $78 \sin(-3.1^\circ)$, about -4.22 m/s . Carrying that downward motion unchanged to the pavement would miss the model’s chosen touchdown target of -0.7 m/s .

The flare has to bend the vertical motion as well as bring the aircraft into contact with the ground.

Height and speed are linked by energy. In the still-air coordinated point-mass model, mechanical energy is $E = \frac{1}{2}mV^2 + mgh$ and its rate of change is $\dot{E} = (\mathcal{T} - D)V$, where $\mathcal{T}$ denotes thrust and D drag. Descending can exchange potential energy for speed; drag dissipates energy and thrust supplies it. Lift changes the direction of the air-relative velocity without doing work along it. A descent calculation therefore needs a speed calculation, and a flare needs both.

The consequences continue after contact. Imagine first a simple ground roll: the aircraft touches down a distance x_D beyond the usable threshold, travels at forward ground speed u for a delay d , then decelerates at a constant rate $a > 0$. In a speed–time diagram the delay contributes a rectangle of area ud . Braking contributes a triangle with height u and base u/a , hence area $u^2/(2a)$. Adding the touchdown station gives

$$S = x_D + \underbrace{ud}_{\text{delay travel}} + \underbrace{\frac{u^2}{2a}}_{\text{braking travel}}, \quad R = L - S. \quad (2)$$

Here S is the stopping station, R the runway remaining and $L = 2786$ m the landing distance available.

The square of speed is the important feature. Holding deceleration fixed, a 10% increase in touchdown ground speed increases the braking distance by 21%, since $1.1^2 = 1.21$. The delay distance also grows, linearly. A few metres per second can spend much more runway than their size suggests when read as a speed difference.

For the constructed landings, the ground law fixes a two-second delay followed by $0.15g = 1.4709975 \text{ m/s}^2$ deceleration, with at least 150 m of runway to remain. These are illustrative research assumptions, not calibrated wet-runway performance. The aircraft model is a reduced coordinated point mass, with finite bank-angle and thrust rates. Its aircraft identity, configuration and estimated mass are anchored to the CAL605 report—a Boeing 747-409B, flaps 30, gear down, estimated 525,718 lb—while its aerodynamic coefficients and operating envelope are stipulated. The entry and corridor are synthetic. This separation lets the calculation expose the coupling among constraints without presenting its path as a recovered historical approach. Appendix A gives the complete equations and constants.

Choosing a path that can carry its own forces

A curve through space determines much more than a picture. Its first derivative gives velocity; the second gives the acceleration that lift and thrust must supply. Changes in acceleration, described by the third derivative or jerk, affect how quickly bank and thrust must change. The model works backwards through these relations: choose a smooth position curve, then derive the forces and rates it would require.

Two endpoint-constrained polynomials describe approach and flare. Twelve parameters determine their durations, touchdown state and interior shape. The approach begins at $(-2500, 1000, 140)$ m in runway-forward, runway-right and vertical coordinates: 2.5 km before the usable threshold, 1 km to its right and 140 m above runway level. Its initial ground track is 47° left of the runway. The threshold target is $(0, 0, 15.24)$ m with runway-aligned ground track, 153 kt equivalent airspeed and a -3.1° air flight-path angle. Equivalent airspeed accounts for the change of air density; the force calculation itself uses airspeed and the model density.

The boundary conditions join approach and flare in position, velocity and acceleration. Control-rate checks use the one-sided derivatives at the join. At touchdown a separate ground-contact rule takes over, with the vertical component reset and the forward motion governed by equation (2). The polynomial construction and its exact parameter ranges are in Appendix B.

What should the optimizer improve? A path that saves runway might demand too much bank; one with a comfortable bank angle might approach a speed or thrust limit. The chosen objective gives each inequality a dimensionless margin and raises the smallest of them. If $g_j(t; q) \geq 0$ is an inequality, $s_j > 0$ its scale and q the trajectory parameters, the primary problem is

$$\max_{q, \eta} \eta \quad \text{such that} \quad \frac{g_j(t; q)}{s_j} \geq \eta \quad \text{for every relevant } t, j, \quad (3)$$

with structural equalities and additional hard constraints enforced separately. A secondary stage reduces integrated squared normalized bank and rate demands while retaining the primary score within 10^{-4} .

The scales specify the tradeoffs. The EAS floor contributes $(V_{\text{EAS,kt}} - 135)/40$, the bank margin is $1 - |\phi|/25^\circ$, and the stop-reserve margin is $(2636 - S)/2636$. Reweighting these expressions can change the preferred path. Thus η is the declared model objective, not a universal measure of aviation safety. The envelope permits bank up to 25° , bank-angle rate up to $5^\circ/\text{s}$ and EAS between 135 and 175 kt, together with lift, load, thrust, thrust-slew and descent limits. A 400 m three-dimensional tube surrounds a fixed synthetic reference cloud. All of these constraints remain active in the construction.

A complete calm-air landing

The retained calm solution reaches the threshold after 37.34 seconds, spends four seconds in the flare and touches down 313.49 m beyond the threshold. Its touchdown ground speed is 77.888 m/s. The two-second delay then uses 155.78 m and the constant-deceleration phase uses another 2,062.05 m. The aircraft model stops at station 2,531.32 m, leaving 254.68 m of runway. From the synthetic entry to rest takes 96.29 seconds.

The small rise early in the path deserves attention. The accepted model limits descent but does not impose monotonic loss of height; the optimizer uses that freedom. The rise stays in both the mathematical reference and its illustration.

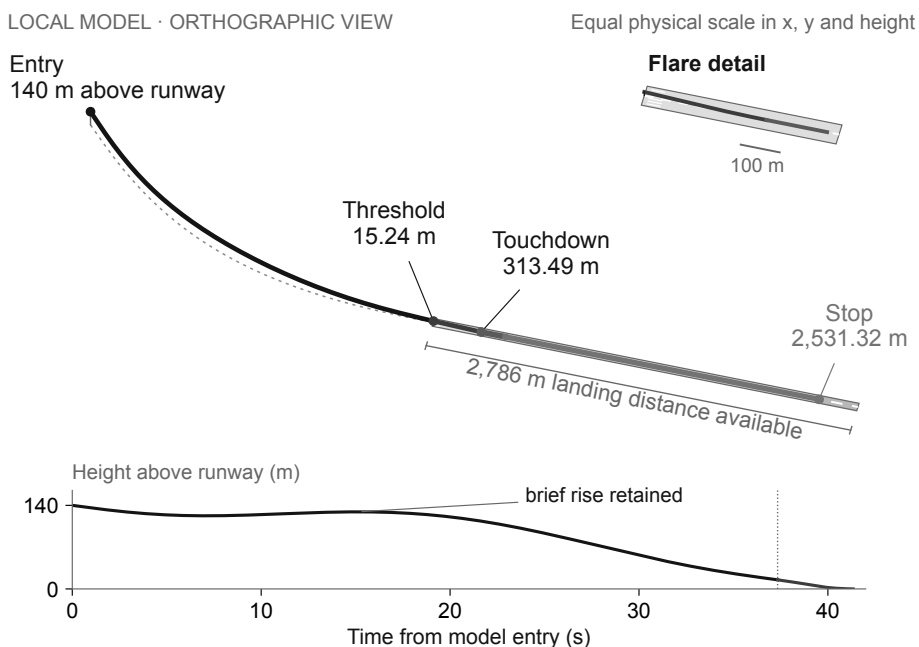


Figure 4

The complete calm-air construction in a local runway frame: approach, flare, touchdown and ground roll. The runway plane, threshold and centreline make the length and height scales explicit; the geometry is not registered onto historical Hong Kong. The source polynomials retain their brief shallow climb. Touchdown is at 313.49 m and the full stop at 2,531.32 m of the 2,786 m landing distance available. Sources: frozen calm reference coefficients and equation (2); ground law: 2 s delay followed by 0.15g.

Removing it to resemble a more familiar approach would change the model, not improve the drawing.

At this touchdown speed, the stopping law has local sensitivities

$$\frac{\partial R}{\partial u} = -d - \frac{u}{a} \approx -54.95 \frac{\text{m}}{\text{m/s}}, \quad \frac{\partial R}{\partial d} = -u \approx -77.89 \frac{\text{m}}{\text{s}}. \quad (4)$$

Another second of delay costs nearly 78 m at fixed touchdown speed. One additional metre per second costs about 55 m locally under the same braking law. Touchdown is therefore a commitment about the use of the runway still ahead. The airborne calculation has to deliver a state from which the ground calculation can succeed.

This reference supplies a complete motion in its declared coordinate system. To connect it to Kai Tak itself, the runway, terrain and buildings would have to occupy that same system with defensible dates and heights. That brings the investigation back to the photographs.

4. Recovering the city around the approach

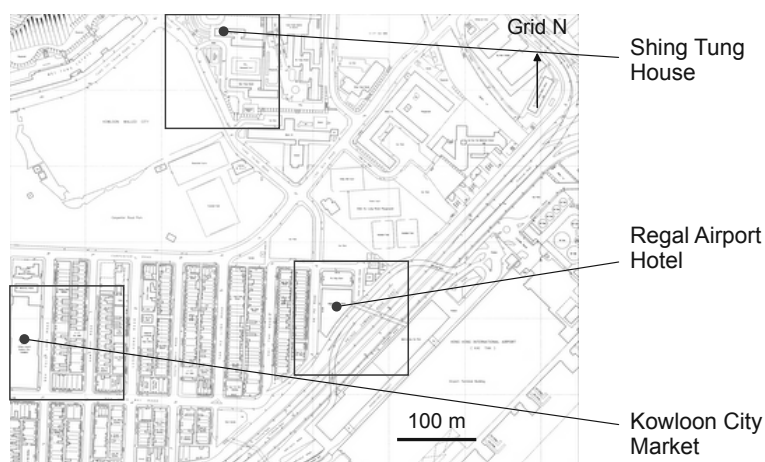
A current map cannot show a vanished building simply by being more detailed. For an approach in 1993, that absence matters: the object missing from today's

image may be the object the aircraft had to pass. The historical problem begins with matching a place to a date.

Survey sheets offer one way in. The October 1989 and July 1994 editions of sheet 11-NE-11A place streets and mapped forms in the same Hong Kong 1980 Grid rectangle. Their labels give recognizable points of reference: Shing Tung House, the Regal Airport Hotel and Kowloon City Market. Viewed together, the editions allow individual forms and their surroundings to be compared across time (Lands Department, Government of the Hong Kong SAR n.d.b). A named form provides a lead; its presence, height and identity at an intervening date require additional evidence.

OCTOBER 1989

Sheet 11-NE-11A



JULY 1994 · the same three map windows



Shing Tung House



Regal Airport Hotel



Kowloon City Market

Figure 5

Kowloon in survey sheet 11-NE-11A: the October 1989 field above, and matched 150 m-wide July 1994 windows below. Named locators let the reader compare mapped forms across editions. The sheets use the Hong Kong 1980 Grid; registered raster spacing is 0.25 m, not a positional-error bound. Matching forms are not automatically accepted as unchanged 1993 buildings. Source: Lands Department, HKSAR Government; CSDI Portal and DATA.GOV.HK, Old Survey Sheets. Government ownership retained; supplied as-is, without endorsement.

A photograph contributes a different kind of evidence. It records visible surfaces together at one exposure. A roof, a road junction and the pattern

of adjoining blocks can be examined in their original setting, without first reducing them to polygons. The overlapping frames A36142 and A36143 are catalogued as 2 November 1993, two days before CAL605; A36143's apparent handwritten 3 November label remains an unresolved discrepancy (Lands Department, Government of the Hong Kong SAR n.d.a).

Depth from a displacement

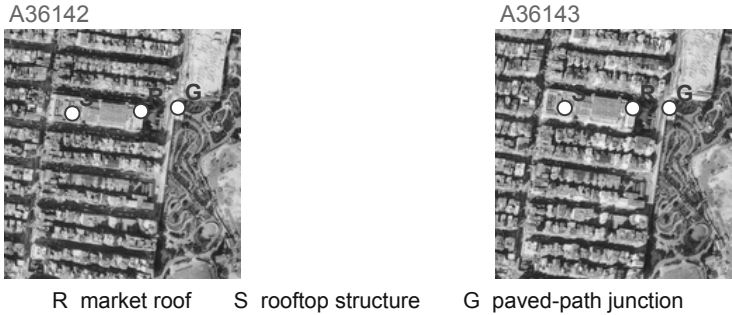
At Kowloon City Market, the roof feature marked R can be picked out in both photographs; S marks a rooftop service structure. Their displacement between the images provides depth information. Once the camera geometry and metric scale are fixed, the corresponding sight lines constrain each feature's position in three dimensions. In the reconstruction, R lies 10.77 m above the fitted local park/road plane and S lies 19.20 m above it. Both examples lie outside the map-control hull: their apparent detail exceeds the direct positional support available at their locations.

Here the image matches are anchored to manually interpreted ground features and survey-road controls. Camera and scan assumptions enter that alignment. The result retains 1,318 sparse matches and 171,307 filtered dense points. The number of points gives a sense of the recovered detail; the distribution of control determines how confidently that detail can be placed.

Only 3,780 dense points lie inside the map-control hull, about 2.21% of the retained cloud. Most of the reconstructed surface therefore extends beyond the small region directly enclosed by the alignment controls. The 2.21% measures that support geometry; it is not an estimate of the fraction of the city reconstructed correctly. A detailed-looking surface can carry much less direct positional support than its appearance suggests.

The held-out horizontal control discrepancy is 5.754 m RMS. That number describes a horizontal registration diagnostic. It supplies no corresponding vertical roof-error bound. The vertical coordinate is conditional on a fitted local park/road plane, so a high point in the rendering means high relative to that plane. It is not automatically a building's height above its own base, still less an absolute roof elevation in a surveyed datum.

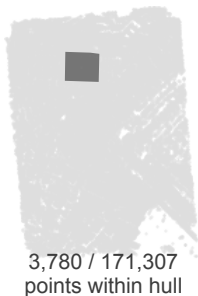
TWO PHOTOGRAPHS ACQUIRE DEPTH



ROOFS AND ROADS, WITH GAPS



CONTROL SUPPORT



True metre scale in all directions; gaps left open.



A mapped identity: Kowloon City Market

R roof feature: 10.77 m above fitted plane

S rooftop structure: 19.20 m above fitted plane

Conditional stereo heights; no absolute roof datum.

Both labelled roof features lie outside the control hull.

Figure 6

From historical photographs to recovered surface structure. The stereo reconstruction derives from overlapping A36142/A36143 images and map controls; the geographic interpretation is tied to the dated survey forms. Heights are relative to a fitted local park/road plane, not HKPD or height above each building’s own ground. The control hull and missing geometry remain visible. The labelled roof features lie outside the control hull. Sources: Lands Department photographs and survey sheets, with the project’s retained stereo/mesh reconstruction. Government/CSDI/DATA.GOV.HK attribution and ownership retained.

A conservative mesh joins the supported geometry while leaving holes and boundaries; it contains 86,061 boundary edges. Those edges mark the limits of the reconstructed surface, including where views or correspondences run out. Closing a hole with a plausible wall would make the scene easier to recognize, but the new wall would come from the illustrator. It would add no observation. The retained mesh therefore remains non-watertight, and its detailed counts and earlier support correction remain in Appendix G.

A height needs a vertical origin

Some survey sheets contain approximate spot elevations above Hong Kong Principal Datum (HKPD). The settled obstacle inventory has 24 such readings at 12 sites on sheet 11-NW-10D, from November 1989 and June 1995. Repeated values can be carried forward between map editions; they are not necessarily independent measurements.

An elevation and a building height answer different questions. If a roof elevation and the underlying terrain already use HKPD, adding the roof number to the terrain number counts the vertical origin twice. Conversely, CAD's runway elevation of 15 ft AMSL, or 4.572 m in that stated reference, cannot simply be set equal to an HKPD elevation. A common vertical transformation still has to be justified. The project's present reconstruction has not accepted one.

Modern terrain and building products help locate potential evidence, but their dates remain attached to them. The DTM uses 2014–2015 photography and a 2019 revision; the building snapshot is from September 2026 (Lands Department, Government of the Hong Kong SAR 2019, 2026). Neither supplies a complete census of what stood around the 1993 approach. Appendix G preserves the full source inventory, dated matching counts and obstacle limitations.

The present historical geometry contains no accepted complete obstacle volumes or physical obstacle upper bounds. Its unknown areas remain marked in the exported scene. To test clearance, those missing pieces matter as much as the visible ones: a real aircraft sweeps wings, tail and landing gear through space, with tracking error around any chosen reference path. The reconstruction has recovered a dated fragment of the city and made its geometric support inspectable. Turning that fragment into a clearance model requires additional observations.

Even a fully recovered city would leave another moving part. The aircraft travels through air that need not move with the ground.

5. The air moves, and the constraints must hold

A wind that stays unchanged over the airport does not keep the same relation to an aircraft turning through it. What begins mostly as a headwind can become a substantial crosswind as the track rotates. The change comes from the geometry of the motion, even before any gust is introduced.

Aerodynamic forces depend on velocity through the air; the runway is fixed to the ground. If $\mathbf{w}$ is a constant wind velocity, the two are related by

$$\underbrace{\dot{\mathbf{r}}}_{\text{ground velocity}} = \underbrace{V\mathbf{e}_V}_{\text{air velocity}} + \underbrace{\mathbf{w}}_{\text{wind velocity}}. \quad (5)$$

The horizontal direction of $\dot{\mathbf{r}}$ is ground track. The horizontal direction of $\mathbf{e}_V$ is air heading. A runway-aligned track can therefore require the aircraft to point

into a crosswind. Equal tracks in two wind conditions need not imply equal headings or ground speeds.

Take the wind scenario's stipulated mean: from 070° true at 21 kt. Meteorological bearings name the direction the wind comes from, so forming its velocity requires reversing that direction. The chart-based convention used here converts 135° magnetic with $1^\circ 50'$ west variation to $133^\circ 10'$ true. In forward/right/up runway coordinates, the wind vector is approximately $(-4.877, 9.640, 0)$ m/s. The original chart's 1990 variation is a scenario convention, not a newly surveyed 1993 runway axis; Appendix D gives the conversion.

ONE WIND · TWO MOVING FRAMES

From 070° true at 21 kt; arrows use a common velocity scale.

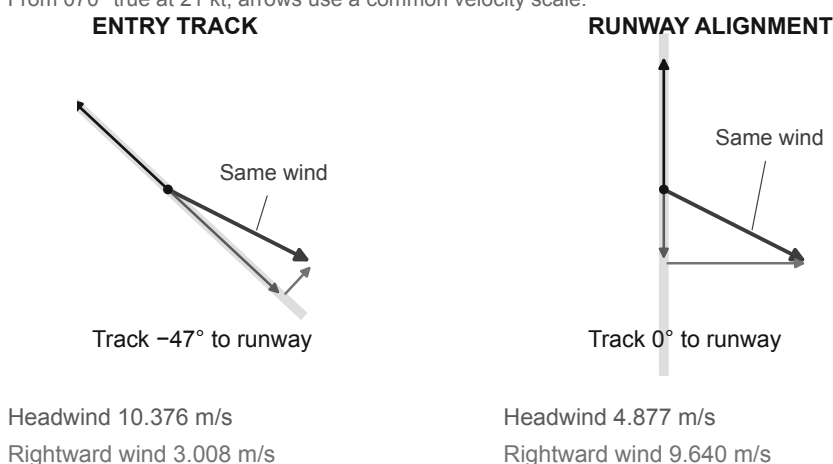


Figure 7

One steady wind, viewed from two track directions. At the -47° entry track its forward and rightward cross-track components are -10.376 and 3.008 m/s; at runway alignment they are -4.877 and 9.640 m/s. The aircraft/track orientations change while the imposed wind stays fixed. The wind is from 070° true at 21 kt, using the chart-based runway bearing $133^\circ 10'$. Source: frozen wind input and trigonometric projection.

At entry, the headwind component is about 10.38 m/s and the rightward cross-track component only 3.01 m/s. At runway alignment, the headwind component has fallen to 4.88 m/s while the rightward cross-track component is 9.64 m/s. A single wind vector has acquired a different role in the landing. A gust field would introduce further changes; this calculation isolates the directional geometry of steady wind.

A second complete construction

The wind-conditioned path is optimized afresh, holding the synthetic entry position, reference corridor, aircraft assumptions, objective and braking law fixed. It crosses the threshold after 41.223 s and touches down after 45.365 s, at

station 302.114 m. Touchdown ground speed is 140.909 kt; equivalent airspeed is 151.558 kt. The former determines the ground distance in the stopping law, while air-relative motion determines the aerodynamic demands.

The full stop comes at 96.644 s and station 2,233.231 m, leaving 552.769 m of runway. Like the calm candidate, the path contains a brief shallow rise permitted by the accepted constraints. The two constructions can now be compared along the same runway scale. Their spatial paths and time profiles appear together in Appendix I.

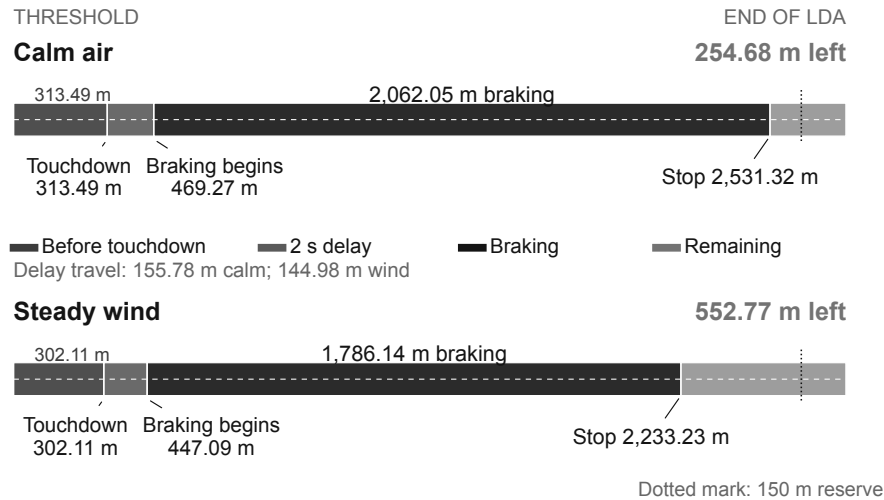


Figure 8
Where the available runway goes. Each row uses the same 2,786 m scale, separating touchdown station, two-second delay travel, braking at 0.15g, and remaining runway. The calm case leaves 254.68 m; the separately optimized steady-wind case leaves 552.77 m. Both wind and the optimized airborne path change, so the difference is not a causal wind effect. Sources: frozen scenario outcomes and equation (2).

The difference in remaining runway is 298.09 m. Of that, 11.38 m comes from the earlier touchdown and 286.71 m from the reduced ground travel. This is an algebraic decomposition of two separately optimized outcomes: it does not isolate the effect of changing wind alone. A controlled comparison would first have to decide what stays fixed, such as the path, endpoint airspeed or control policy. Here the question is whether each declared case admits a complete construction under the common modelling rules.

The narrow failure a plot can hide

There is one more difficulty in answering that question. A numerical optimizer usually examines constraints at selected times. Between two accepted samples, a margin could briefly become negative. A curve can look smooth and a plot can look feasible while missing the very interval that matters.

THE SPACE BETWEEN THE SAMPLES

Each shaded rectangle encloses every curve value in its interval.

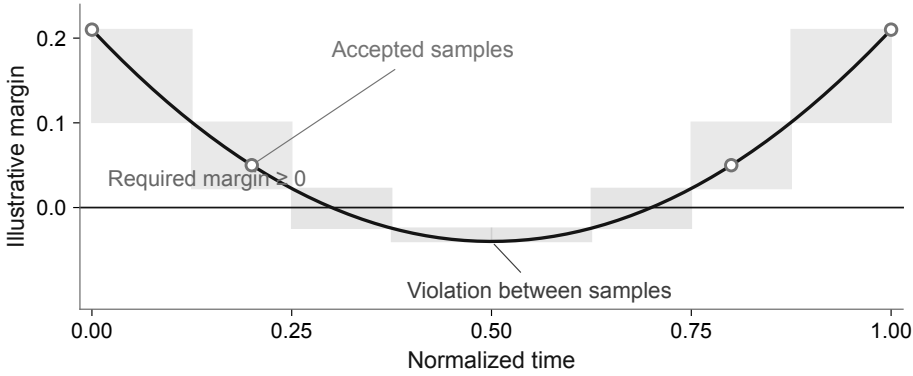


Figure 9

Why checking samples and checking a whole interval answer different questions. The explanatory curve illustrates how positive sample values can miss a violation between them; interval enclosures cover the values throughout each time span. This pedagogical construction is separate from the two certified landing references. The actual calm and wind checks cover 2,502 and 376 closed intervals respectively, including both sides of the approach–flare join.

Interval arithmetic replaces a single value at a time point with an enclosure over a closed span of time. For these fixed polynomial references, the checker carries those enclosures through the inverse dynamics, forces and control rates, rounding outwards so arithmetic rounding does not shrink the claimed range. If an enclosure is too wide to settle a constraint, the time interval is subdivided. The accepted intervals must cover the complete approach and flare, with no gaps. Endpoint identities and the exact ground rule are checked separately.

The calm reference has a certified normalized margin of at least 0.0397, using 2,502 accepted intervals. The separate wind certificate establishes at least 0.1507934696816132, using 376 intervals. Each binds its own exact real-decimal candidate and the specified reduced model. The different interval counts reflect the candidates and numerical enclosures, not a ranking of safety or proof strength. Appendix D records the mpmath implementation and reproduction procedure.

Feasibility supplies a lower bound on the best attainable score in each family. Necessary restrictions give upper bounds:

$$0.0397 \leq \eta_{\text{calm}}^* < 0.1559772,$$

$$0.1507934696816132 \leq \eta_{\text{wind}}^* \leq \frac{66656817}{307536544} \approx 0.2167443782.$$

These brackets concern different wind-conditioned feasible sets. The wind upper bound exceeds its certified lower bound by about 0.06595091. Neither bracket identifies the optimum or establishes useful near-optimality; the upper

bounds relax requirements that a complete flight still has to meet. Their derivations appear in Appendices C and D.

The declared constraints hold throughout the two constructed landings, including intervals too small to read from a plotted line. An actual landing brings additional states, measurements and decisions into the problem. The historical flight record preserves some of them.

6. What the runway record reveals

On 4 November 1993, China Airlines flight CAL605, Boeing 747-409B B-165, overran Runway 13 and entered the sea. A tropical cyclone affected the airport; the flight encountered strong gusty easterly wind, rain, a wet runway and windshear during the visual turn. The official investigation followed the landing through its flight record, weather observations and rollout sequence (Hong Kong Civil Aviation Department 1995).

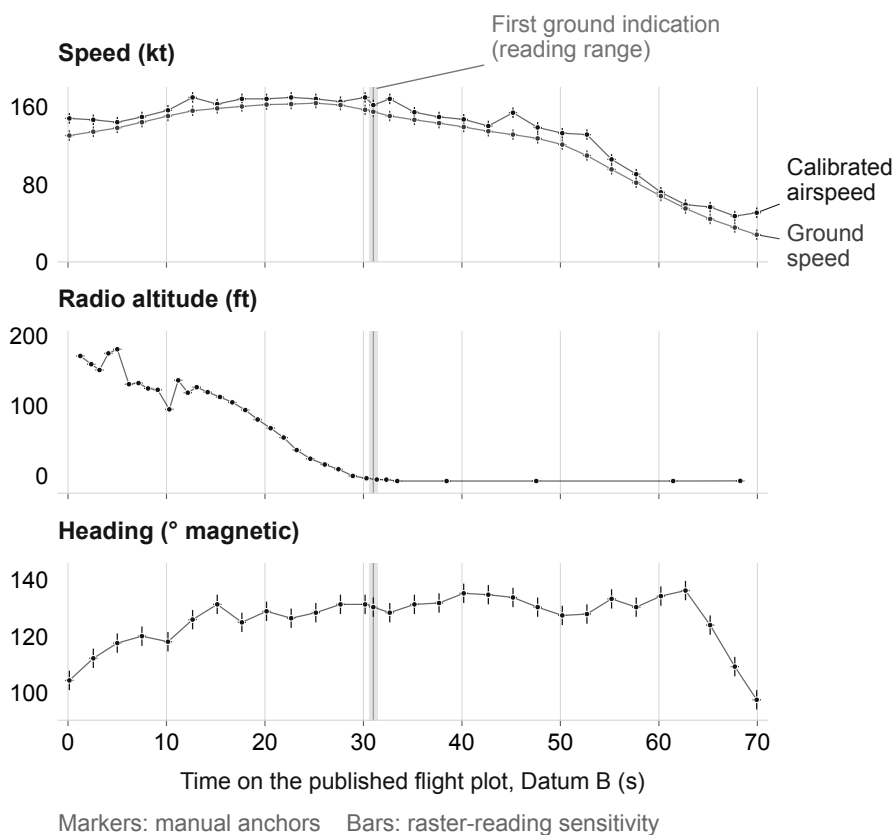
Its finding about touchdown is the starting point here. The touchdown was controllable, and its parameters were not causal in the accident (Hong Kong Civil Aviation Department 1995, §2.6.5). The most conspicuous part of the approach was not, in the investigation's account, the part that determined the outcome.

The report's major causes concerned rollout actions and their operational setting. The commander inadvertently advanced the thrust levers while intending to select reverse. His instructions and limited communication reduced the co-pilot's monitoring of rollout and autobrake operation. The investigation also identified the co-pilot's inadequate skill and experience for controlling the rollout in the strong gusty crosswind, and the absence of a clearly defined crosswind-landing technique in the airline's manual (Hong Kong Civil Aviation Department 1995, Synopsis; §3). These findings describe an interacting aircraft system and crew, far beyond the two numbers in a constant-deceleration law.

The reported sequence included forward thrust, autobrake disarming, spoiler retraction and delayed reverse thrust, with stronger deceleration later. Each transition changes what happens after contact. A stopping model with one delay and one constant brake rate cannot recover that sequence merely by refining the touchdown position. The historical record supplies the missing mechanism.

Reading a recorder through a printed page

The report describes 269 recorded parameters but publishes selected plots, not the raw numerical stream. Recovering values from the publication means reading curves through line thickness, overlapping traces and printed axes. The retained benchmark has 190 selected approach/final-phase pixel anchors and 35 speed-distance anchors. Figure 10 uses the final-phase readings directly, with their raster-reading sensitivity.

**Figure 10**

Selected readings from the published CAL605 recorder plot, CAD Report 1/95, Appendix 11, Figure 4. Markers are manually extracted anchors; connecting lines guide the eye. Bars indicate raster-reading sensitivity, not sensor accuracy or physical confidence intervals. The shaded strip marks the first ground indication's declared reading range, not exact physical touchdown time or sensor uncertainty. CAS and ground speed are distinct quantities; radio altitude is not height above a level runway throughout the approach. Heading is magnetic. The horizontal axis uses the plot's Datum B, separate from the ground-weather clock.

Some absences cannot be repaired by finer digitization. Thrust-lever angle was not recorded. Cockpit wheel, column and pedal positions are not automatically aerodynamic control-surface deflections. The report's Appendix 12 track was reconstructed by integrating acceleration, so it is not an independent positional observation against which those same accelerations can be validated.

Even touchdown distance has several source conventions. The narrative reports about 2,300 m remaining; Appendix 13 gives about 2,250 m; the digitized ground-flag transition is around 2,210 m. Combining the 2,786 m LDA with the rounded 480 m touchdown station gives 2,306 m. These quantities retain their separate origins. Their differences identify a reconstruction task, not an invitation to average them into a falsely precise location. Appendix G preserves the measurement and clock conventions.

The wind had more than one reading

The airport wind printout contains 15 irregular timestamps, five sensors and three fields, giving 225 direction–speed pairs. Field A is the maximum one-second speed within a five-second window; B and C are overlapping two- and ten-minute averages. At 03:35:30 on the wind system’s clock, the northwest sensor reports 37 kt in A, 23 kt in B and 19 kt in C. All three describe the wind, but at different temporal aggregations.

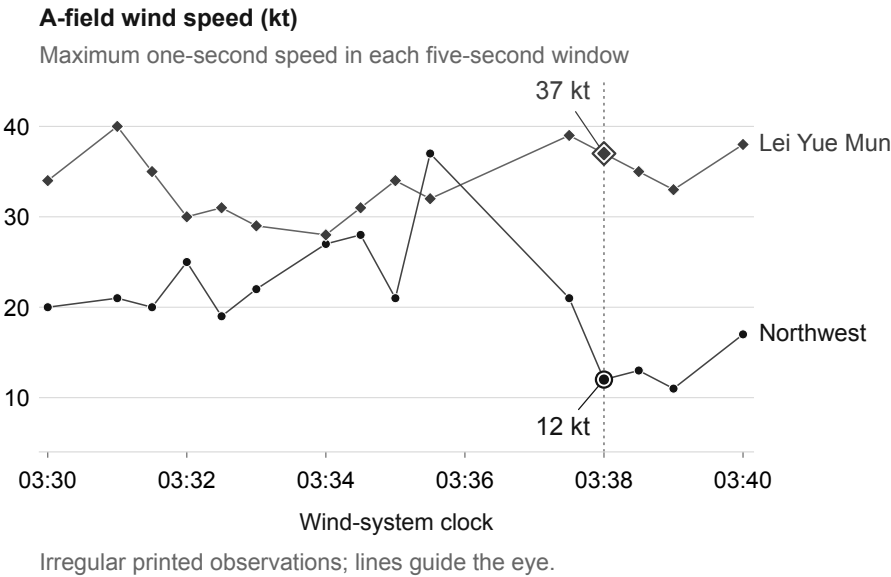


Figure 11
Ground-wind observations in their own measurement frame. The northwest and Lei Yue Mun readings are A-field values: maximum one-second speed within a five-second window, printed at irregular event-triggered times. At 03:38:00 the two sensors report 12 and 37 kt. The clock is the wind system’s, with no accepted offset to the aircraft timeline. Source: CAD Report 1/95, Appendix 7, retained transcription. These ground observations do not recover aircraft-local wind.

At 03:38:00, A-field readings at the northwest and Lei Yue Mun sensors are 12 and 37 kt. The spatial difference is large, but it is a difference between ground instruments; it does not measure a wind gradient encountered by the aircraft. Event-triggered printing also leaves peaks between entries unobserved in this table. The offset between the wind-system clock and the flight/ATC timeline remains unidentified, preventing an exact gust-to-flight-event alignment.

The NOAA archive adds wider context. Its 03:00 and 04:00 UTC observations give 070° true at 10.8 m/s, consistent with the report’s rounded 21 kt mean; gust fields are 17.5 and 20.6 m/s (NOAA National Centers for Environmental Information n.d.b,a). In the declared 8,360-row routine subset for 1993, 8,285 speeds are lower than 10.8 m/s, 17 tie and 58 are higher. This locates the reported mean within that selected archive. Dependence between

records, incomplete coverage and selection prevent turning the rank into a return period or landing-risk probability. The archive and report may also share originating observations. Agreement between them need not be independent corroboration.

Slower, yet with a harder stop ahead

A speed trace falls during the rollout. That alone does not tell us whether the remaining stop is becoming easier. The relevant comparison is between speed squared and the runway still available.

If speed is v , remaining runway is R and another delay τ precedes constant braking, the deceleration needed to stop at the end is

$$a_{\text{req}} = \frac{v^2}{2(R - v\tau)}, \quad R > v\tau. \tag{6}$$

A decrease in v helps the numerator. Loss of runway reduces the denominator. The second can outrun the first.

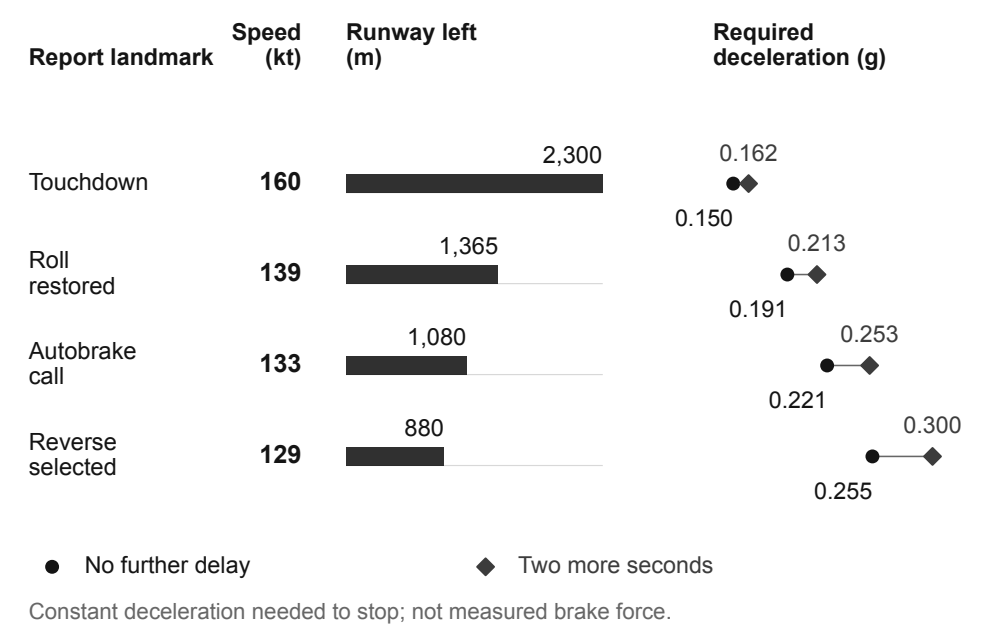


Figure 12

Four report-stated rollout landmarks, translated into the constant deceleration still required by equation (6). The calculation distinguishes an immediate constant-deceleration stop from one delayed by two seconds. These are conditional stopping requirements, not measurements of applied brake force or a continuous deceleration trace. Sources: CAD Report 1/95, §§2.2.5–2.2.7; retained landmark calculations.

Across the four reported landmarks, the speed falls from 160 to 129 kt while the zero-additional-delay stopping requirement rises from approximately 0.150g to 0.255g. At the last point, another two seconds raises the requirement

to roughly 0.300g. The aircraft is slowing, but the available runway is disappearing faster than the falling speed can compensate.

The same distinction protects the interpretation of the earlier benchmark. Inserting the report's rounded 480 m touchdown station and 160 kt ground speed into the fixed two-second/0.15g law gives a remainder of -161.52 m. That is an extrapolated scalar outcome under the selected braking assumptions, not a reconstruction of movement into the sea. It does not overturn the investigation's finding that a normal landing roll remained possible at touchdown. Nor would a constructed path with more reserve establish accident prevention.

Touchdown passes a position and speed to a ground system whose state can change. Completing the landing requires both enough pavement and the appropriate sequence of braking actions. The mathematical reference makes the first dependence explicit; the accident record shows why the second cannot be compressed into a constant without consequences.

7. What the landing requires

The 47-degree mismatch is easier to understand after following the motion to its end. A modest bank accumulates a substantial turn because lift acts over time. Radius and turning rate set the space and time needed. Descent and flare must then deliver a touchdown state from which the remaining runway can absorb the forward motion.

The two constructed landings put these relationships into one calculation. Each joins an approach, a flare and a full stop, with a separate continuous-time certificate. They expose choices hidden by a simple drawing: the synthetic entry and corridor, the allowance for a small rise, the scaling of competing constraints and the stipulated braking law. An explicit path allows those choices to be examined individually.

Within the restricted calm-air family, compactness and regularity also establish that a best primary member exists, together with a secondary effort minimum on the retained-primary set. The proof in Appendix C establishes attainment without identifying the optimizers. The remaining gaps between achieved scores and necessary upper bounds concern this mathematical family; they do not establish that the historical procedure was suboptimal.

Putting the calculation back into the city

The historical images make the next challenge tangible. The recovered surface has roof forms, roads and openings, but its vertical origin remains local and much of its geometry lies outside the control hull. A clearance calculation needs a common runway/obstacle datum, dated terrain and roof coverage, physical upper bounds, and the aircraft's swept volume with a defensible tracking allowance. A more complete rendering helps only when additional geometry comes from additional evidence.

The aircraft and braking model must also be calibrated to the intended configuration and conditions, then tested against routine flights. Uncertain wind requires a causal control policy using information available at the time; choosing a different curve after each wind is known does not supply one. Go-around viability and physically supported outcome uncertainty remain part of that larger task. Appendix H gives the full research requirements and related methods, while Appendix F derives the conditional stopping bounds.

The opening question was how a large aircraft could negotiate the turn and still have enough space, time and runway to land. The calculations explain how those demands fit together. Bank redirects lift; sustained turning accumulates direction; descent changes the energy balance; wind separates air motion from ground motion; touchdown commits the remaining pavement to delay and braking. Each stage constrains the next.

Under the declared constraints, turn, descent and stop fit within one continuous mathematical construction. Connecting that construction to a historical landing requires the dated city and a calibrated aircraft. Kai Tak drew the eye to the turn. The calculation keeps following the aircraft until it stops.

Appendix A. Dynamics and exact control rates

Let $\mathbf{r} = (x, y, h)$, $\mathbf{v} = \dot{\mathbf{r}} - \mathbf{w}$, $V = \|\mathbf{v}\|$, $\gamma = \arcsin(v_h/V)$ and $\psi = \text{atan2}(v_y, v_x)$ for constant wind. Define the orthonormal frame

$$\mathbf{e}_V = (\cos\gamma\cos\psi, \cos\gamma\sin\psi, \sin\gamma), \quad (7)$$

$$\mathbf{e}_\gamma = (-\sin\gamma\cos\psi, -\sin\gamma\sin\psi, \cos\gamma), \quad (8)$$

$$\mathbf{e}_\psi = (-\sin\psi, \cos\psi, 0), \quad \mathbf{f} = \ddot{\mathbf{r}} + (0, 0, g). \quad (9)$$

Writing $\ell_\gamma = \mathbf{f} \cdot \mathbf{e}_\gamma$ and $\ell_\psi = \mathbf{f} \cdot \mathbf{e}_\psi$, inverse dynamics gives

$$\mathcal{L} = m\sqrt{\ell_\gamma^2 + \ell_\psi^2}, \quad \phi = \text{atan2}(\ell_\psi, \ell_\gamma), \quad (10)$$

$$Q = \frac{1}{2}\rho V^2 S_w, \quad C_L = \mathcal{L}/Q, \quad (11)$$

$$D = Q(C_{D0} + k_D C_L^2), \quad \mathcal{T} = m\mathbf{f} \cdot \mathbf{e}_V + D, \quad (12)$$

$$\rho = \rho_0 e^{-h/H_\rho}, \quad V_{\text{EAS}} = V\sqrt{\rho/\rho_0}. \quad (13)$$

Here S_w is wing area. For the time derivatives used in the checker,

$$\dot{\ell}_i = \ddot{\mathbf{r}} \cdot \mathbf{e}_i + \mathbf{f} \cdot \dot{\mathbf{e}}_i, \quad \dot{\phi} = \frac{\ell_\gamma \dot{\ell}_\psi - \ell_\psi \dot{\ell}_\gamma}{\ell_\gamma^2 + \ell_\psi^2}, \quad (14)$$

$$\dot{Q} = Q(-\dot{h}/H_\rho + 2\dot{V}/V), \quad \dot{C}_L = \dot{\mathcal{L}}/Q - C_L \dot{Q}/Q, \quad (15)$$

$$\dot{D} = \dot{Q}(C_{D0} + k_D C_L^2) + 2Qk_D C_L \dot{C}_L, \quad \dot{\mathcal{T}} = m(\ddot{\mathbf{r}} \cdot \mathbf{e}_V + \mathbf{f} \cdot \dot{\mathbf{e}}_V) + \dot{D}. \quad (16)$$

Differentiating the three frame vectors supplies the remaining derivatives. The inverse construction requires positive total speed, horizontal airspeed and lift; the interval checker verifies this nonsingular branch. Its bank-angle rate is the rate of ϕ in the moving flight-path frame, not generally the body-axis roll rate.

Forward replay integrates

$$\dot{\mathbf{r}} = V\mathbf{e}_V + \mathbf{w}, \quad \dot{V} = (\mathcal{T} - D)/m - g\sin\gamma, \quad (17)$$

$$\dot{\gamma} = (\mathcal{L}\cos\phi/m - g\cos\gamma)/V, \quad \dot{\psi} = \mathcal{L}\sin\phi/(mV\cos\gamma). \quad (18)$$

A replay that recomputes lift from the replayed state and C_L command checks force consistency. It remains numerical corroboration, not continuous-time proof or physical calibration. If wind varies along the motion, air-relative acceleration also involves $-d\mathbf{w}/dt$; the constant-wind equations cannot be extended by changing only the position equation.

Frozen constants. $m = 238461.67357166$ kg is the exact conversion of an estimated 525,718 lb, not an exact measured mass; $g = 9.80665$ m/s², $\rho_0 = 1.225$ kg/m³, $H_\rho = 8500$ m, $S_w = 541.2$ m², $C_{D0} = 0.08$, $k_D = 0.045$. Low-altitude CAS is approximated by EAS for the 153 kt boundary speed. These choices do not establish a calibrated aircraft envelope.

Table A1

The frozen research envelope. Source: model configuration. These are stipulated limits; the reported aircraft identity and estimated mass do not calibrate them.

Quantity	Constraint
Bank / bank-angle rate	$ \phi \leq 25^\circ; \dot{\phi} \leq 5^\circ/\text{s}$
Equivalent airspeed	135–175 kt
Lift coefficient / load factor	$0 \leq C_L \leq 2.8; 0.5\text{--}1.4$
Thrust / magnitude of thrust slew	0–700 kN; at most 100 kN/s
Descent / approach sink	No steeper than 6° ; at most 8 m/s before flare
Touchdown station	200–700 m beyond usable threshold
Touchdown: lateral / track / bank	5 m; 2° ; 3°
Touchdown sink	0–1.5 m/s; constructed target 0.7 m/s
Reference corridor	400 m three-dimensional distance from a fixed reference cloud
Ground rule / reserve	2 s delay, then 0.15g; at least 150 m remaining

Appendix B. Explicit polynomial construction

For a phase of duration $T > 0$, put $s = (t - t_0)/T \in [0, 1]$. Given endpoint positions p_0, p_1 , velocities v_0, v_1 and accelerations a_0, a_1 , construct the vector-valued quintic

$$c_0 = p_0, \quad c_1 = T v_0, \quad c_2 = T^2 a_0 / 2, \quad (19)$$

$$A = p_1 - c_0 - c_1 - c_2, \quad B = T v_1 - c_1 - 2c_2, \quad C = T^2 a_1 - 2c_2, \quad (20)$$

$$c_3 = 10A - 4B + C/2, \quad c_4 = -15A + 7B - C, \quad c_5 = 6A - 3B + C/2. \quad (21)$$

Then $P(s) = \sum_{k=0}^5 c_k s^k$ satisfies the six endpoint conditions by substitution and differentiation. To change the interior while preserving those conditions, use

$$B_0(s) = 64s^3(1-s)^3, \quad B_1(s) = (2s-1)B_0(s).$$

Each function and its first two derivatives vanish at both endpoints. Coordinate-specific multiples produce the degree-at-most-seven approach and degree-at-most-six flare. Time derivative order j is $T^{-j} d^j/ds^j$; omitting these duration factors would change the dynamics.

The restricted parameter family

The original box has $T_A \in [30, 55]$ s, $T_F \in [4, 12]$ s, $x_D \in [200, 700]$ m and $u \in [69, 88]$ m/s. Four horizontal approach amplitudes lie in $[-400, 400]$ m and two vertical amplitudes in $[-50, 50]$ m. Flare amplitudes lie in $[-50, 50]$ m horizontally and $[-10, 10]$ m vertically. These twelve bounded parameters define the family; they are not all possible flight controls.

The entry is $(-2500, 1000, 140)$ m and the threshold state $(0, 0, 15.24)$ m. Entry/threshold EAS is 153 kt and air flight-path angle -3.1° . The selected initial ground track is -47° ; threshold track is zero. Touchdown is $(x_D, 0, 0)$ with ground velocity $(u, 0, -0.7)$ m/s. Endpoint accelerations are zero;

approach and flare meet in position, velocity and acceleration. Contact then resets the vertical component under a separate ground rule.

For a horizontal ground-track unit vector $\mathbf{d} = (\cos \chi, \sin \chi)$ and horizontal airspeed $H = V \cos \gamma$, endpoint forward ground speed solves $\|u_g \mathbf{d} - \mathbf{w}_{xy}\|^2 = H^2$:

$$u_g = \mathbf{d} \cdot \mathbf{w}_{xy} + \sqrt{H^2 - \|\mathbf{w}_{xy}\|^2 + (\mathbf{d} \cdot \mathbf{w}_{xy})^2}.$$

The positive branch requires a nonnegative radicand. Thus equal EAS, flight-path angle and ground track across wind scenarios do not imply equal ground velocity.

The coefficients printed in Appendix E are rounded evaluation copies. Machine-readable exports preserve full floating precision. The rigorous margin claims instead bind exact real-decimal parameter vectors and reconstruct the endpoint expressions with interval arithmetic; they do not certify arbitrarily rounded coefficient tables.

Appendix C. Attainment and the calm-family bound

The retained computer-assisted regularity lemma establishes $|v_h| < 14$ m/s throughout both phases over the full parameter box. Here v_h means vertical velocity. On the nonnegative-height feasible domain, $\text{EAS} \geq 135$ kt implies total airspeed at least 135κ m/s, where $\kappa = 1852/3600$. The vertical bound then keeps horizontal speed above 68 m/s. Positive minimum load keeps lift nonzero, and the bank bound retains a positive-up lift projection.

Proposition 1 (Restricted calm-family attainment). In the frozen zero-wind polynomial family, the primary maximum is attained. The specified continuous secondary effort has a minimum on the set retaining that optimum within 10^{-4} .

Proof. The certified candidate makes the set with $\eta \geq 0.0397$ nonempty. Restrict q to its compact parameter box and η to a closed bounded interval containing the valid bounds. Along a convergent feasible sequence, polynomial states and derivatives through jerk converge uniformly, since durations stay positive. The regularity bounds exclude singularities at the limit, so inverse forces, rates and margins are continuous there. Distance to the fixed finite corridor cloud is continuous. Every time-indexed inequality and endpoint equality passes to the limit. The feasible set is therefore compact, and projection onto η attains its maximum. Continuous effort also attains a minimum on the nonempty closed retained-primary subset. ■

This attainment statement concerns the specified calm polynomial family. Identifying the maximizing parameters is a separate problem; neither the wind family nor unrestricted aircraft controls inherit this proof.

At touchdown the normalized EAS floor implies

$$u_{\min}(\eta) = \sqrt{[\kappa(135 + 40\eta)]^2 - 0.7^2} \leq u.$$

In the zero-wind flare, lateral motion is zero, forward velocity is positive, and $|v_h| < 14$. Hence $v_x \geq \sqrt{[\kappa(135+40\eta)]^2 - 14^2}$. Since $T_F \geq 4$ s, integrating from threshold supplies a lower bound on x_D . Combining it with $S \leq 2636(1-\eta)$ gives

$$4\sqrt{[\kappa(135+40\eta)]^2 - 14^2} + 2u_{\min}(\eta) + \frac{u_{\min}(\eta)^2}{2(1.4709975)} \leq 2636(1-\eta). \quad (22)$$

The left side minus the right side increases strictly for $\eta \geq 0$. Outward-rounded evaluation brackets its unique nonnegative root between 0.1559771 and 0.1559772. Therefore $\eta_{\text{calm}}^* < 0.1559772$. This necessary bound relaxes airborne restrictions and need not be attainable.

The corrected checker uses a full angle hull when interval `atan2` crosses its problematic branch and never uses a lower corridor-margin enclosure as an exclusion upper bound. The earlier purported upper limits 0.10, 0.05, 0.04 and 0.03972 were withdrawn after these defects were found. A later 15,000-box search left 7,071 boxes unresolved at 0.10; its budget exhaustion supplies no stronger bound. Source: `analytical/coupled/RESULTS.md`, proof code and frozen verification records.

Appendix D. Wind convention and separate upper bound

Let wind speed be W , its meteorological *from* bearing β , and runway true bearing α . In forward/right/up axes,

$$w_x = -W \cos(\beta - \alpha), \quad w_y = -W \sin(\beta - \alpha), \quad w_h = 0.$$

Use $W = 21(1852/3600)$ m/s, $\beta = 70^\circ$, and the scenario convention $\alpha = 135^\circ - 1^\circ 50' = 799/6$ degrees. The source chart's 1990 variation is not a surveyed 1993 runway axis. The report's rounded 21 kt is the frozen input; NOAA's 10.8 m/s is not silently substituted.

The wind-case upper bound follows from only three necessary restrictions. Normalized touchdown position requires $x_D \geq 200 + 500\eta$; the original parameter box gives $u \geq 69$ m/s; normalized stopping margin requires

$$x_D + 2u + \frac{u^2}{2a} \leq 2636(1-\eta), \quad a = 1.4709975 \text{ m/s}^2.$$

Since the ground-travel expression increases with nonnegative u ,

$$200 + 500\eta + 138 + \frac{4761}{2a} \leq 2636(1-\eta), \quad (23)$$

$$\eta \leq \frac{2636 - 200 - 138 - 4761/(2a)}{3136} = \frac{66656817}{307536544}. \quad (24)$$

The exact rational upper bound minus the exact decimal lower certificate is

$$\frac{1584555818464679384939}{240262925000000000000000} \approx 0.0659509085084467.$$

A relaxation can give a valid upper bound even when no feasible flight realizes its limiting touchdown speed and position together. This is why the bound is necessary, not a candidate or an optimum. The sharper-looking calm bound uses a zero-wind airspeed/forward-speed relation and cannot simply be reused.

Certificate scope and reproducibility

The wind checker encloses inverse dynamics, exact rates and hard inequalities over both phases; the ground rule is evaluated separately. The source binds the model configuration, original corridor cloud, candidate, calculator and checker by hashes. The calm and wind certificates cover 2,502 and 376 accepted closed intervals, respectively. Certification decisions compare outward-rounded intervals, not their displayed floating summaries.

The companion `verify_certificates.py` re-evaluates both fixed witnesses using frozen original proof code and writes new receipts without rerunning optimization or overwriting source certificates. The manuscript builder independently checks source identity, time-cover completeness, polynomial/CSV agreement and scalar arithmetic. These complementary checks establish reproducible computer-assisted evidence for the specified mathematical objects, not an independent peer review or validated aircraft performance.

Appendix E. Numerical evaluation coefficients

For each coordinate, $r_i(t) = \sum_k c_{ik} s^k$, $s = (t - t_0)/T$. All coefficients below have units of metres and are rounded to nine decimal places. Do not extrapolate outside a phase. For exact figure reproduction use the JSON exports; for certification use the source-bound real-decimal parameter construction. Approach starts at $t_0=0$; flare starts at $t_0=T_A$.

Calm-air witness**Approach:** $T = 37.3377606433$ s.

k	c_{xk} (m)	c_{yk} (m)	c_{hk} (m)
0	-2500.000000000	1000.000000000	140.000000000
1	2017.911295685	-2163.944933098	-160.244035284
2	0.000000000	0.000000000	0.000000000
3	3413.929294279	4100.690248399	3438.022240773
4	-10204.999953354	-4778.257389771	-9897.948219630
5	14739.635372583	1189.804675387	9803.845767246
6	-10064.642384084	1536.071423529	-3418.719289702
7	2598.166374892	-884.364024447	110.283536597

Flare: $T = 4.0000000000$ s.

k	c_{xk} (m)	c_{yk} (m)	c_{hk} (m)
0	0.000000000	0.000000000	15.240000000
1	314.661243433	0.000000000	-17.041443788
2	0.000000000	0.000000000	0.000000000
3	-1.795376483	0.000000000	-51.839797579
4	3.395313292	0.000000000	111.333830611
5	-5.309847192	0.000000000	-70.581049549
6	2.540848108	0.000000000	12.888460304
7	0.000000000	0.000000000	0.000000000

Steady-wind witness**Approach:** $T = 41.2226012074$ s.

k	c_{xk} (m)	c_{yk} (m)	c_{hk} (m)
0	-2500.000000000	1000.000000000	140.000000000
1	1934.549368102	-2074.550210351	-176.916768670
2	0.000000000	0.000000000	0.000000000
3	3276.552981363	3383.062356916	2933.950323176
4	-8656.604149697	-3364.647218569	-8305.273504155
5	11688.322856040	-86.179330521	8399.649065340
6	-7637.663260946	2181.352151193	-3255.475138584
7	1894.842205137	-1039.037748667	279.306022894

Flare: $T = 4.1419425532$ s.

k	c_{xk} (m)	c_{yk} (m)	c_{hk} (m)
0	0.000000000	0.000000000	15.240000000
1	303.172858064	0.000000000	-17.646170298
2	0.000000000	0.000000000	0.000000000
3	-2.193352218	0.000000000	-36.770067274
4	5.314434930	0.000000000	72.668703740
5	-7.478355627	0.000000000	-35.336994379
6	3.298466890	0.000000000	1.844528212

Appendix F. Ground motion, bounds and reproduction

With $q_g = t - t_D$, ground motion after the stipulated contact transition is

$$x(t) = \begin{cases} x_D + uq_g, & 0 \leq q_g \leq d, \\ x_D + ud + u(q_g - d) - \frac{1}{2}a(q_g - d)^2, & d \leq q_g \leq d + u/a. \end{cases}$$

It remains constant afterwards. Lateral and vertical ground coordinates are zero. The model does not resolve landing-gear impact or braking-system modes. Integrating speed gives equation (2); the full-stop time is $T_A + T_F + d + u/a$. Because stopping station increases with x_D, u, d and decreases with a , the extrema in equation (26) follow directly. Likewise a joint box gives

$$t_{\text{stop}} \in [A_- + F_- + d_- + u_-/a_+, A_+ + F_+ + d_+ + u_+/a_-].$$

For a joint Cartesian input box with $a_- > 0$ and nonnegative speed and delay, the stopping extrema are

$$S_- = x_- + u_-d_- + \frac{u_-^2}{2a_+}, \quad S_+ = x_+ + u_+d_+ + \frac{u_+^2}{2a_-}, \quad (25)$$

$$R \in [L_- - S_+, L_+ - S_-]. \quad (26)$$

Monotonicity gives exact extrema over this box without a probability distribution or a statistical-independence assumption. If some extreme combinations cannot occur together, or violate airborne constraints, the enclosure can be conservative for the smaller physically admissible set. No physical input box has been calibrated for these examples.

A prediction interval for an individual landing requires a justified conditional outcome distribution and validation in its intended domain. A confidence interval for a mean or parameter answers a different question. Marginal endpoint intervals do not, on their own, give joint coverage of an entire path or of obstacle clearance.

Reproduction map

File or directory	Role
<code>main.tex</code> , <code>sections/</code>	Editable narrative and layout; this appendix contains derivations.
<code>references.bib</code>	Bibliographic records and primary-source links.
<code>make_figures.py</code>	Reproducible V5 illustrations from frozen numerical inputs and evidence-linked archival sources.
<code>inputs/</code> , <code>evidence/</code>	Dated permitted imagery, derived observations, coefficients, proof code and source reports.
<code>build.py</code>	Hash checks, numerical checks, figures, compilation, text/font/layout checks and page rendering.
<code>verify_certificates.py</code>	Fresh continuous-time checks of both fixed witnesses.
<code>CLAIMS_AND_EVIDENCE.csv</code>	Claim categories, exact source paths, limitations and verification routes.
<code>VERIFICATION.md</code> , <code>qa/</code>	What was checked, actual results, visual inspection record and remaining scientific gaps.

The build README distinguishes a manuscript rebuild from proof verification and historical-data acquisition. The compact package reproduces the displayed numerical results and fixed-witness checks. It does not recreate every earlier source acquisition, stereo reconstruction or optimization search from scratch; the wider repository preserves their code and larger-data manifests.

All important statements are assigned an evidence category: source-reported, digitized approximation, reconstructed geometry, model assumption, numerical construction, computer-assisted certificate, theorem within a stated domain, or unresolved. The V5 changes concern exposition and visual communication. Versions 2–4 and all frozen model inputs remain preserved.

Appendix G. Historical sources and their measurement conventions

The historical reconstruction and the landing calculations use different coordinate systems. The model’s origin is the usable Runway 13 threshold; x runs forward, y to the right and h upward from the runway plane. Its entry point and corridor are stipulated. Map-derived horizontal coordinates use the Hong Kong 1980 Grid, EPSG:2326. Stereo elevations are relative to a fitted local park/road plane. A transformation among these frames sufficient for historical clearance has not been accepted.

Runway and photographic dates

CAD Report 1/95 records 3,331.5 m of runway length, 61 m width and 2,786 m landing distance available; the stopping law uses the last quantity. The official airport history's 3,390 m extension-era length is a different reported quantity (Hong Kong Civil Aviation Department 1995, n.d.). The working accident date is 4 November 1993. A36142/A36143 are catalogued as 2 November; A36143's handwritten label appears to read 3 November. This discrepancy is retained. Additional colour frames CN4581 and CN4589 are dated 8 October 1993; no seamless same-day orthophoto is implied by displaying the frames together.

Map inventory and incomplete surface geometry

The underlying holdings contain 227 valid survey-map archives covering 93 core sheet locations and 48 full aerial frames. Four road-cell comparisons yielded 404 dated assignments of 202 current templates, 531 dated map-region assignments and 397 cross-date overlaps. These are correspondence candidates, not 935 verified historical buildings. The dated-form inventory contains 45 reviewed forms for 15 named forms.

The retained stereo model has 1,318 sparse matches and 171,307 filtered dense points. Its conservative mesh uses 155,891 vertices and 225,371 triangles, with 86,061 boundary edges. The accepted mesh incorporates the earlier 49-point interpolation-support correction; the invalid predecessor remains excluded. The 3,780 dense points inside the map-control hull constitute 2.2066% of the dense set. The 5.754 m held-out RMS discrepancy measures horizontal fit, not vertical roof error or a worst-case positional bound.

On sheet 11-NW-10D, the November 1989 and June 1995 editions supply 24 approximate mapped surface elevations at 12 sites. The legends identify metre elevations above Hong Kong Principal Datum (HKPD). Repeated values can be carried-forward cartography rather than independent surveys. A top elevation above HKPD is not a building height to add to a terrain elevation expressed in the same datum. The report's 15 ft AMSL runway elevation converts to 4.572 m in its stated reference; that conversion does not reconcile AMSL with HKPD.

The modern 5 m DTM derives from 2014–2015 photography, with a March 2019 revision. The building dataset used in the wider project is a September 2026 snapshot (Lands Department, Government of the Hong Kong SAR 2019, 2026). These modern products are useful contextual sources, not substitutes for the 1993 city. The accepted historical obstacle inventory still has zero complete obstacle volumes, zero physical obstacle upper bounds and `historical_clearance_ready=false`. Unknown-coverage polygons are retained, so missing evidence never becomes an assertion of clear airspace.

The CAL605 plots and clocks

The report describes 269 recorded parameters but publishes selected plots, not the raw numerical stream. Thrust-lever angle was not recorded. Cockpit wheel, column and pedal positions are distinct from aerodynamic surface deflections. Appendix 12's reconstructed track was obtained by integrating accelerations, so it cannot independently validate those same accelerations.

The retained digitization has 190 selected approach/final-phase pixel anchors and 35 speed–distance anchors. Their reading ranges describe uncertainty in locating traces on a raster, not sensor accuracy. Touchdown-distance conventions remain distinct: about 2,300 m remaining in the narrative, about 2,250 m in Appendix 13, roughly 2,210 m at the digitized ground-flag transition, and 2,306 m from the threshold-consistent subtraction 2786–480. No average is used to conceal those differences.

The wind printout has 15 irregular timestamps, five sensors and three fields, giving 225 direction–speed pairs. Field A is the maximum one-second speed within a five-second window; B and C are overlapping two- and ten-minute averages. Printing is event-triggered, so absent rows are not evidence of absent peaks. The wind-system clock offset to the flight/ATC timeline is unidentified.

NOAA ISD station-year 450070–99999 contains 8,549 rows for 1993, including 24 on 4 November. The specified routine quality/format subset contains 8,360 rows: at 10.8 m/s, 8,285 are lower, 17 tie and 58 are higher (NOAA National Centers for Environmental Information n.d.b,a). Serial dependence, selection and incomplete coverage preclude turning that rank into a return period or landing-risk probability. NOAA and the report may share originating Hong Kong observations.

Appendix H. Scope, open research questions and source rights

What the certificates establish

Each certificate encloses a fixed exact real-decimal construction under the declared reduced point-mass equations. The implementation uses mpmath interval arithmetic at 40 decimal digits, not proof-assistant formalization. Both sides of the approach–flare join, control rates, endpoints and the exact stopping law are checked. Rounded coefficient tables, plotted samples and small forward-replay residuals are complementary evaluation aids, not replacements for the certified object.

The calculations establish feasible references and necessary bounds within specified finite-dimensional families. Neither global optimizer is identified. The calm attainment argument establishes existence, including the continuous secondary minimum on the retained-primary set; it supplies no formula for the attaining parameters. The invalidated stronger exclusions described in Appendix C remain outside the accepted evidence.

What historical validation would require

Historical flyability requires a supported entry state, a common horizontal and vertical reference, dated obstacle coverage, a calibrated aircraft envelope and a swept-volume calculation for wings, tail and landing gear. Point-mass clearance alone would not meet this requirement. Operational use would additionally require tracking and observation-error bounds, a causal feedback policy, routine-flight validation and go-around viability. None of these missing inputs is supplied by the scenario margin.

The information order matters. A robust-control claim would require

$\exists \pi \quad \forall w \in \mathcal{W}: \quad \text{the causal policy satisfies the required constraints,}$

whereas separate known-wind constructions have the weaker form

$\forall w \in \mathcal{W} \quad \exists \pi_w: \quad \text{some separately chosen solution works for that wind.}$

The two computed cases establish neither a result for an entire wind set nor the existence of the first policy. A fair comparison with a historical procedure also needs a supported historical baseline and equivalent aircraft, environmental and information conditions.

The official report’s causal conclusions govern the CAL605 discussion. Its controllable-touchdown finding is not contradicted by a negative remainder under an assumed constant-braking formula. An accident-prevention counterfactual would require the operational braking and crew/system states absent from that formula.

Related methods and uncertainty objects

Steady-wind Dubins-airplane planning studies geometry under restricted motion; richer six-degree-of-freedom landing models represent additional aircraft states; tracking frameworks such as FaSTrack explicitly connect planning to error bounds (Wu and Chen 2024; Kim et al. 2024; Chen et al. 2021). Each addresses a particular part of the problem. Adapting those ideas here requires checking their assumptions against this aircraft, these observations and this historical setting. Their algorithms and guarantees have not been independently reproduced or transferred by this investigation.

A raster-reading range, a horizontal registration residual, a forward-replay residual and an objective bound describe different objects. None supplies a physical prediction interval for a landing. The exact joint-box stopping extrema in Appendix F become physically informative only after the input set has been justified and linked to the airborne constraints.

The fixed-braking CAL605 calculation’s loss of remainder relative to the calm witness comprises 166.51 m from the later touchdown and 249.69 m from the speed change. This is an algebraic comparison under the same two-second/0.15g rule, not an attribution of actual accident causes.

Attribution and reproducibility

Photographs, survey sheets and derived historical geometry are sourced from the Lands Department, Government of the Hong Kong Special Administrative Region, through the CSDI Portal and DATA.GOV.HK. Government and relevant organisations retain intellectual-property ownership; the material is supplied as-is, without endorsement. Open-data reproduction is subject to the cited attribution terms (Government of the Hong Kong SAR 2025; Development Bureau, Government of the Hong Kong SAR n.d.). Annotations and interpretations are the author's. The permission is not generalized to paid imagery or other institutions' documents.

CAD report facts and the project's identified manual readings are replotted; report scans and scholarly PDFs without established redistribution permission are not included in the portable package. Source-specific terms accompany the inputs. No blanket original-work licence is granted by this edition. The RFS author class and supporting files retain their own notices; proprietary installable fonts are not redistributed.

The companion reports distinguish a clean manuscript/figure rebuild, fresh fixed-witness verification and the wider historical acquisition programme. Original permitted inputs, derived observations and model scenarios remain separate. This organization lets a later correction change a particular claim without silently rewriting the evidence behind an earlier edition.

Research assistance. Aryan Ayyar initiated the project and directed its scientific scope. Research assistance, implementation, numerical checks, figure generation and drafting used AI tools. The foreword remains for author approval; no independent human specialist review or institutional endorsement is implied.

Appendix I. Companion figures and source context

The common local-coordinate view below retains both exact polynomial references. The profiles separate timing from geometry: paths that look almost coincident in space can be traversed at different speeds. The brief rises are present in both accepted constructions.

TWO CONSTRUCTED LANDINGS

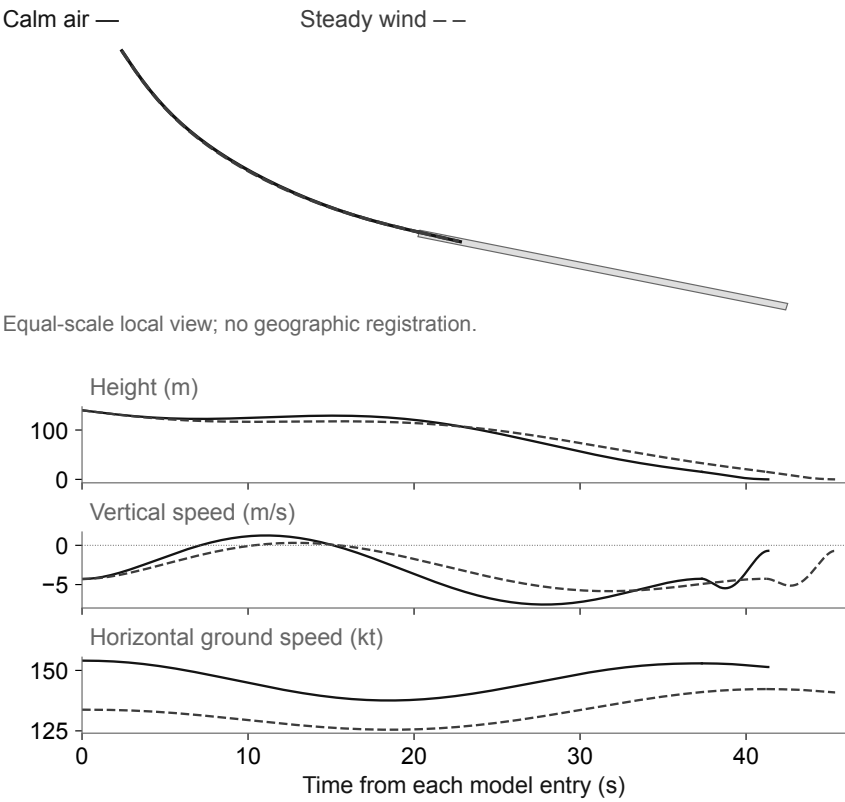


Figure A1
Calm and steady-wind paths evaluated directly from their frozen coefficients. Height is relative to the runway plane; vertical speed is positive upwards; horizontal ground speed is the magnitude of the horizontal ground-velocity vector. Each curve uses time from its own model entry. The orthographic scene has equal metric scale in all three directions. The paths are separate scenario solutions, not recorded flights.

WIND-CONDITIONED CONTROL HISTORY

Dashed rules: model limits. Vertical rule: approach–flare join.

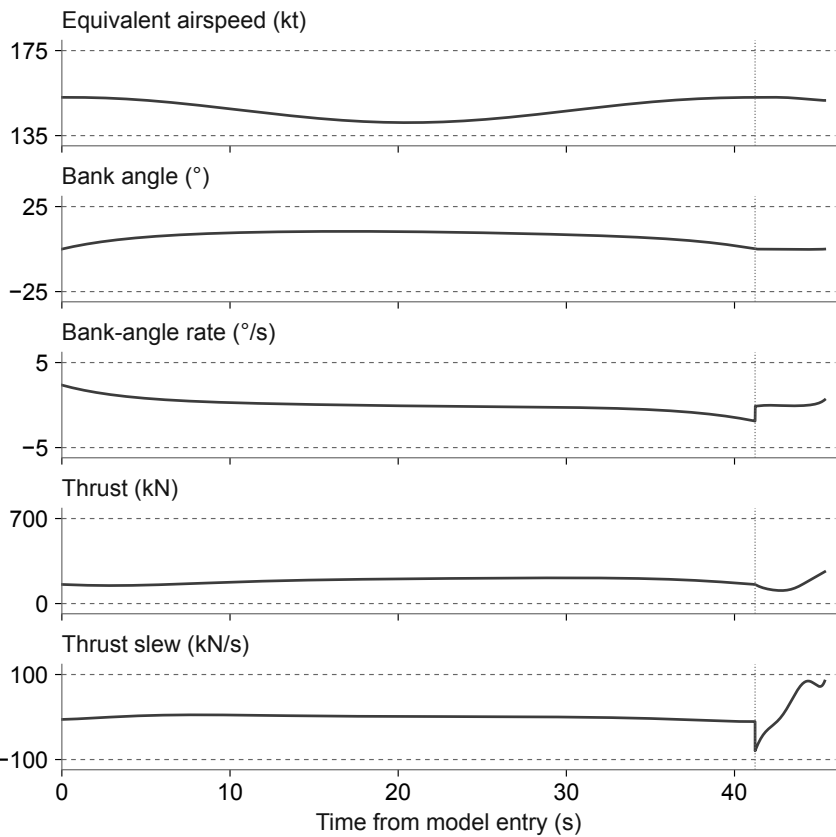


Figure A2

Control histories for the wind-conditioned reference, from the retained path/control table. Model limits are shown as dashed horizontal rules; the vertical rule marks the approach–flare join. Bank-angle rate is measured in the moving flight-path frame. The display is sampled; continuous validity follows from the separate interval certificate, including one-sided join checks.

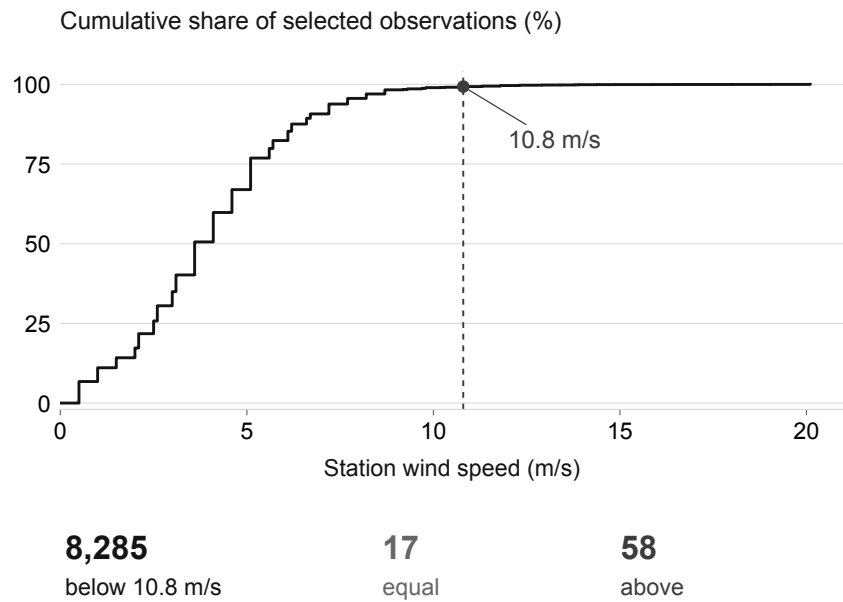


Figure A3
Annual station context from the specified 1993 NOAA subset. The empirical distribution contains 8,360 qualifying routine reports: 8,285 below 10.8 m/s, 17 equal and 58 above. Repeated weather reports are not independent extreme-wind events. This distribution is neither an accident-risk estimate nor a reconstruction of conditions along the aircraft path.

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